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THIRD ORDER ABERRATIONS IN THE SLAC 50 GEV COLLIDER

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Recently Richter[1,2] has suggested an interesting proposal for the modification of the SLAC facility to provide collisions of bunches of 50 GeV positrons and electrons. That proposal presents many interesting challenges in accelerator design and in this note we discuss one of these challenges.

High luminosity for the SLAC 50 GeV collider requires focussing of electron and positron beam bunches to extremely small radii ($\lesssim$ 1 micron) at the interaction area with a large momentum acceptance. It is necessary to correct second order chromatic aberrations in order to obtain this spot size with a large momentum acceptance, and Brown has designed a correcting transport system using sextupoles and dipoles which eliminates these second order aberrations.[1,3]

The focussing requirements of the SLAC collider suggest that higher order aberrations may also be important, and in this note we estimate the size of third order aberrations in the SLAC collider. The focussing problem is the same as the problem of focussing for heavy ion fusion (small target size), so that the methods used to calculate third order aberrations for heavy ion fusion (HIF) may be used to estimate their size in the 50 GeV SLAC collider.

II. Focussing Parameters

To estimate final focussing design for the SLAC collider we will use beam and target parameters described informally by Richter (some of which may be misremembered). We have an interaction point beam radius r_T of 1 micron and an emittance ε of 2×10^{-9} m.-R so that the beam focussing angle ($\Theta = \varepsilon/r_T$) is 2×10^{-3} radians.

Since we do not have a detailed design of the SLAC collider, we assume that the major final focussing elements will be either a doublet or triplet followed by a drift space before the interaction region. Since triplet geometric aberrations are larger, we will use a doublet lens, so our focussing system is FDO (see figure 1).

Garren has calculated doublet focussing parameters in scaled variables,[4] and we can use his tables to estimate the quadrupole doublet length and the magnet bore radii for a number of focussing cases. In these tables, focussing is parallel to point, the beams are symmetric ($\varepsilon_x = \varepsilon_y$) and the target spot is round. In Table I we list focussing parameters (quadrupole lengths and bore radii) for two pole tip fields (conventional 2T and superconducting 4T) and two drift lengths (5 m. and 10 m.). These cases will give us sample focussing systems for estimates of the effects of third order aberrations.

III. Geometric Aberrations in the SLAC 50 GeV Collider

In this section we will estimate the size of "geometric" aberrations; that is, third order aberrations that are independent of the particle energy. The third order equations of motion of a particle in a quadrupole system have been derived previously by Meads[5] and others.[6] The result is that the simple transverse (x-y) harmonic motion of a particle:

$$\frac{d^2x}{dz^2} \equiv x'' = -\phi(z)x \qquad \text{where } \phi = \frac{B'}{B\rho} \tag{1}$$

is modified to

$$\frac{d^2x}{dz^2} = x'' = -\phi(z)\,x + \left(-\frac{3}{2}x'^2x - \frac{1}{2}y'^2x + x'y'y\right)\phi(z) + x\,y\,y'\,\phi'(z) + \left(\frac{y^2x}{4} + \frac{x^3}{12}\right)\phi''(z) \tag{2}$$

with a similar equation for y" where the extra terms arise from necessary quadrupole fringe field effects and geometric factors.

A Green's function integration may be used to find the third order correction to the first order motion. The correction to x is

$$\Delta x = \int_0^{z_F} G_x(z_F,\xi) \cdot \left\{\left(-\frac{3}{2}x'^2 - \frac{1}{2}y'^2x + x'y'y\right)\phi + xyy'\phi' + \left(\frac{y^2x}{4} + \frac{x^2}{12}\right)\phi''\right\} d\xi \tag{3}$$

where $G_x(z_F,\xi) = X_o(z_F)X_e(\xi) - X_e(z_F)\,X_o(\xi)$ and X_o, X_e are the odd and even solutions of the unperturbed equations. At $z = 0$ and $z = z_F$, $\phi = \phi' = \phi'' = 0$.

We can integrate by parts to obtain

$$\Delta x = \int_0^{z_F} \phi^2\, G_x(-xy^2 - \tfrac{1}{3}x^3)\; d\xi \qquad \text{A} \tag{4}$$

$$+ \frac{1}{2}\int_0^{z_F} \phi\, G_x'\, x'\,(x^2+y^2)\; d\xi \qquad \text{B}$$

$$+ \int_0^{z_F} \phi\; G_x\;(-x(x'^2 + y'^2) + x'yy')\; d\xi \qquad \text{C}$$

A rough estimate of the size of geometric aberrations can be obtained by evaluating equation 4 for an extreme ray. The dominant term in (4) is A, which we can evaluate for an extreme ray with initial coordinates given by ($x(0) = R_Q$, $x'(0) = 0$, $y(0) = y'(0) = 0$). For parallel to point focussing $x_o(z_F) \cong L_o$, and $X_e(z_F0 = 0$ and $X_o(0) = 0$, $X_e(0) = 1$. Placing these values in equation 5 and integrating by multiplying by the magnet length we obtain:

$$|\Delta x_{max}| \cong \frac{1}{3} \phi^2 \quad x(0)^3 \cdot L_o \cdot (L_F + L_D)$$

$$\cong \frac{1}{3} \frac{B'^2}{(B\rho)^2} \; R_Q^{\;3} \cdot L_o \cdot (L_F + L_D)$$

For case A ($B = 2T$, $L_o = 5$ m.) we obtain $|\Delta x_{max}| = 8.4\mu$, which is significantly greater than the 1μ radius spot size. In Table II we tabulate this approximate value of $|\Delta x_{max}|$ for our focussing cases. A more accurate estimate of Δx_{max} can be obtained using reference 6, in which Δx_{max} has been calculated for parallel to point focussing as a function of the Garren parameter $b = \frac{BL}{B\rho\Theta}$. The result can be expressed in terms of a single scaled function y(b):

$$|\Delta x_{max}| \cong \quad g(b) \; L_o \; \Theta_o^3$$

The resulting values of $|\Delta x_{max}|$ are tabulated in Table II for cases A - D.

In these cases we see that $|\Delta x_{max}|$ may vary from a few microns to tens of microns. We also see that weaker fields B and shorter drift distances L_o are favored. In fact, superconducting magnets (4T) lead to larger geometric aberrations in this parameter region.

A smaller value of $|\Delta x_{max}|$ (by a factor of ~ 2) may be obtained by optimizing the doublet system; that is, reducing the drift length until it is similar to the doublet length. We will not attempt optimization in this note, since this would require a more precise knowledge of the parameters and constraints of the SLAC Collider. The entire transport system will affect the geometric aberrations.

Calculation of the effect of geometric aberrations on machine luminosity requires integration of the overlap of distorted beam profiles at the interaction point, which we will not attempt in this note. A rough estimate of the luminosity dilution may be obtained from the calculations of beam size distortion at the final focus of reference 6. For the cases of Table I, this dilution would range from a factor of ~10 for case A to ~ 400 for case D. The dilution is not as large as may be suggested by the large values of $|\Delta x_{max}|$, because of the cubic dependence of third order aberrations, but it is significant.

Our estimates suggest that for a high luminosity collider it may be necessary to include third order correction elements in the collider transport design, such as the sets of octupoles suggested by Fenster[7] for HIF transport. In any case the estimates indicate that third order aberrations must be considered in the collider design, and their effects minimized.

IV. A Comment on Third Order Chromatic Aberrations

A simple first estimate of the size of chromatic aberrations can be made by considering the thin lens focus of figure 2. The focussing strength is k, L_o the drift length and R, r_T are the beam sizes at the lens and at the interaction point. k is a function of the momentum:

$$k = k_o \left(1 + \alpha\frac{\delta p}{p} + \beta\left(\frac{\delta p}{p}\right)^2 + \ldots\right)$$

An estimate of the allowable momentum spread can be found by requiring that the extreme ray deviate from the target by less than r_T. Thus,

$$R\left(\alpha\,\frac{\delta p}{p} + \beta\left(\frac{\delta p}{p}\right)^2 + \ldots\right) \lesssim r_T$$

The first term is the second order chromatic aberration term. If second order aberrations are not corrected, we have a limit:

$$\frac{\delta p}{p} \lesssim \frac{1}{\alpha}\frac{r_T}{R} \lesssim \frac{r_T^{\,2}}{\alpha\,\varepsilon L}.$$

With $r_T = 1 \times 10^{-6}$ m., $L = 5$ m., $\varepsilon = 2 \times 10^{-9}$ m-R and the dimensionless parameter $\alpha = 1$, we have a momentum acceptance of 1×10^{-4}. In the 50 GeV colliding beams proposal a much larger momentum acceptance is desired, and sextupole correcting systems, as suggested by Brown, must be used to correct second order chromatic aberrations.

The same approximate analysis can be applied to the third order term (containing β). In matrix form we have

$$\Delta x \sim U_{1166}\;\; x_o \left(\frac{\delta p}{p}\right)^2$$

The requirement $\Delta x \lesssim r_T$ and $\beta \cong 1$ gives us a limit

$$\frac{\delta p}{p} \lesssim 1 \times 10^{-2}$$

which indicates that if a momentum spread greater than about one per cent (1%) is desired, third order chromatic aberrations must be corrected.

CONCLUSION:

We have estimated the magnitude of third order aberrations in the final focus of the SLAC 50 GeV collider and have found them to be significantly large. These aberrations will become increasingly important if high luminosities are required ($r_T < 2$ microns, $\frac{\delta p}{p} > 1\%$). The design of a complete collider transport system compensating for third order aberrations will be a significant challenge.

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TABLE I: FDO Parameters for Final Focussing in the SLAC 100 GeV Collider

CASE	L_o (m.)	B_p (T.)	$b = \frac{BL_o}{B\rho\Theta}$ (Garren parameter)	L_F (m.)	L_D (m.)	R_Q (cm)
A	5	2	30.	0.84	0.98	1.9
B	5	4	60.	0.49	0.53	1.6
C	10	2	60.	0.98	1.06	3.2
D	10	4	120.	0.58	0.63	2.8

TABLE II: Geometric Aberrations in Test Cases

CASE	$\lvert\Delta x_{max}\rvert$ (approximate)	g(b)	$\lvert\Delta x_{max}\rvert$ (reference 6)
A	8.4 (microns)	187	7.5 μ
B	15.7	330	13.2
C	31.4	330	26.4
D	64.7	670	53.6

REFERENCES

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3. K. L. Brown, Proceedings of the 1977 Heavy Ion Fusion Workshop, BNL 50769, p. 107(1977).

4. A. Garren, ERDA Summer Study of Heavy Ions for Inertial Fusion, LBL-5543, p. 102 (1976).

5. P. F. Meads, Jr., Ph.D. Thesis, UCRL-10807, May 15, 1963.

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Fig. 1 – Doublet Focussing Parameters

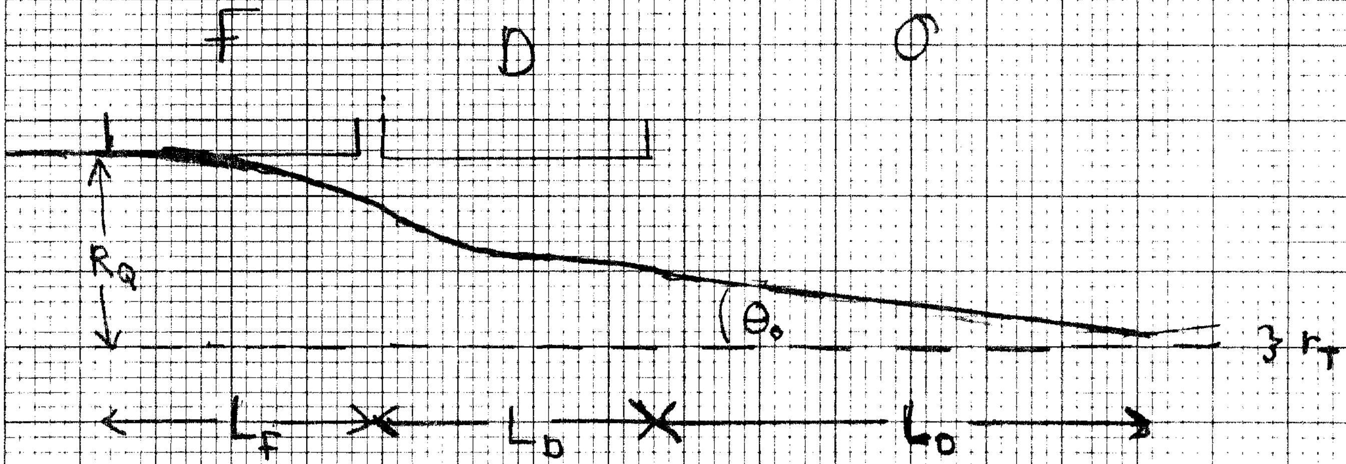
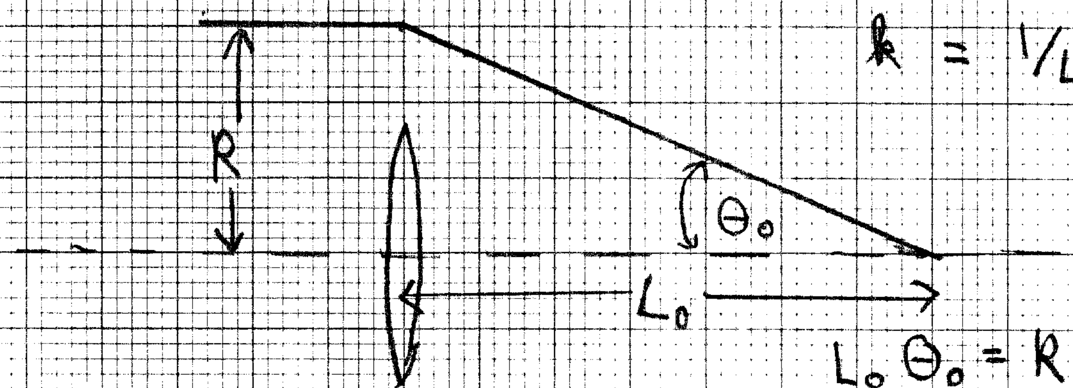


Fig. 2 – Singlet (thin lens) Focus

$\theta_0 = \epsilon / r_T$

$k = 1/L_0$



$L_0 \theta_0 = R$